On the Transformations of Variables in the Solution of Compressible Boundary Layer Equations

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This paper is concerned with the compressible fluid flow, i.e. the dissociated gas in the boundary layer. It defines in a logical way the transformations of the longitudinal and transversal variables of the boundary layer, which are already known and found in the literature. These transformations are used as preceding ones to General Similarity Method for the solution of compressible boundary layer equations.

This paper shows that these transformations can be defined from the condition of the identity of the form of the corresponding values and boundary layer equations of compressible and incompressible fluid.

Keywords: *boundary layer, compressible fluid, transformations of the variables, momentum equation, general similarity method.*

1. INTRODUCTION

While solving different problems of fluid flow in the boundary layer, investigators, as known [1], use different transformations of variables. Instead of the physical coordinates x and y new variables are introduced in the boundary layer theory, usually in the form of the transformations $\xi = \xi(x)$ and $\eta = \eta(x, y)$. Firstly, special transformations are introduced in the boundary layer theory by means of which, using the stream function, the governing partial differential equations are brought down to simple differential equations. This way we come to the so-called similar or ″self-similar″ solutions of the boundary layer equations [2], which are relatively less important for engineering practice.

With the progress of science and engineering, methods for solution of more and more complicated problems of boundary layer fluid flow were developed. Finally, after the so-called parametric methods [2], the General Similarity Method was developed and it was used to solve very complicated problems of compressible fluid flow [3]. This method was successfully used for the solution of MHD boundary layer flow [4] as well as for the dissociated and ionized gas boundary layer flow [5].

With the application of General Similarity Method to the case of the compressible fluid flow, besides general similarity transformations, we use transformations of different forms [2, 3] as previous. First, we use them to transform the starting boundary layer equations, then we use general similarity transformations which enable introduction of the corresponding set of similarity parameters of the considered problem. In the literature, concerning this question [6, 7] newly introduced variables in the form of transformations are used as the preceding ones for the solution of the problem of dissociated or ionized gas flow

$$s(x) = \frac{1}{\rho_0 \mu_0} \int_0^x \rho_w \mu_w \mathrm{d}x \ , \quad z(x, y) = \frac{1}{\rho_0} \int_0^y \rho \, \mathrm{d}y \ . \qquad (1)$$

Often, with the application of the General Similarity Method, there are questions such as: ″Why do these transformations [1] have these forms?″, and ″What is the purpose of introduction of these variables?″ The main goal of the analysis here undertaken is to answer these questions.

2. DEFINING OF THE NEW VARIABLES

In order to answer the questions we start from the continuity equation and the corresponding dynamic equation of the compressible boundary layer, i.e. dissociated gas [6, 7]. These equations with the corresponding boundary conditions are:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \ ,$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{\mathrm{d}u_e}{\mathrm{d}x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \ , \qquad (2)$$

$$y = 0: \qquad u = 0, \qquad v = v_w(x) \ ,$$

$$y \to \infty: \qquad u \to u_e(x).$$

With the equations of the system (2), the usual notations in the theory of 2-D laminar boundary layer [8] are used for different physical values. Here $u(x, y)$ is the longitudinal projection of the velocity in the boundary

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layer, $v(x,y)$ is the transversal projection, ρ is the density of the compressible fluid, and μ is the dynamic viscosity. In the equations (2) and in the boundary conditions, the subscript $''e''$ stands for the physical quantities at the outer edge of the boundary layer, while the subscript $''w''$ represents the values of the physical values at the wall of the body within the fluid. In this analysis we observe the gas flow along the porous wall of the body within the fluid. That is why $v_w(x)$ stands for the given velocity with which the dissociated gas flows perpendicularly through the solid porous wall. As known, this velocity can be $v_w > 0$ at injection or $v_w < 0$ at ejection of the gas.

If, by the usual procedure, the continuity equation is multiplied with $u_e(x)$, we obtain the equation

$$\frac{\partial}{\partial x}(\rho u u_e)+\frac{\partial}{\partial y}(\rho v u_e)=\rho u\frac{\mathrm{d}u_e}{\mathrm{d}x} .$$

The dynamic equation of the starting system (2) can be written in the following form

$$\frac{\partial}{\partial x}(\rho u u)+\frac{\partial}{\partial y}(\rho v u)=\rho_e u_e\frac{\mathrm{d}u_e}{\mathrm{d}x}+\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}) .$$

Subtracting the previous equations we get

$$\begin{aligned}&\frac{\partial}{\partial x}(\rho u u_e-\rho u u)+\frac{\partial}{\partial y}(\rho v u_e-\rho v u)=\\&=\frac{\mathrm{d}u_e}{\mathrm{d}x}(\rho u-\rho_e u_e)-\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y}).\end{aligned} \tag{3}$$

If we multiply the obtained equation with $\mathrm{d}y$ and integrate each term with respect to the variable y in the range from 0 to ∞, i.e. transversally to the boundary layer, we get

$$\begin{aligned}&\int_0^\infty\frac{\partial}{\partial x}(\rho u u_e-\rho u u)\mathrm{d}y+\int_0^\infty\frac{\partial}{\partial y}(\rho v u_e-\rho v u)\mathrm{d}y=\\&=\int_0^\infty\frac{du_e}{dx}(\rho u-\rho_e u_e)\mathrm{d}y-\int_0^\infty\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})\mathrm{d}y .\end{aligned}$$

Taking the boundary conditions into consideration as well as the rules for changing the order of operations of differentiation and integration, the previous equation can be written as

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^\infty\rho u(u_e-u)\mathrm{d}y\right]-\rho_w v_w u_e=\\&=\frac{\mathrm{d}u_e}{\mathrm{d}x}\int_0^\infty(\rho u-\rho_e u_e)\mathrm{d}y+(\mu\frac{\partial u}{\partial y})_{y=0} .\end{aligned} \tag{4}$$

The last term of Eq. (4) represents the shear stress τ_w at the wall of the body within the fluid, i.e.

$$I_4=\tau_w=(\mu\frac{\partial u}{\partial y})_{y=0} .$$

In order to give an answer to the asked questions, we will consider certain terms of Eq. (4), bearing in mind that $u=u(x,y)$, $\rho=\rho(x,y)$ and $u_e=u_e(x)$. The first term of Eq. (4) can be written as:

$$I_1=\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_0^\infty\rho u(u_e-u)\mathrm{d}y\right]=\frac{\mathrm{d}}{\mathrm{d}x}\left[u_e^2\int_0^\infty\rho\frac{u}{u_e}(1-\frac{u}{u_e})\mathrm{d}y\right] \tag{5}$$

From the boundary layer theory of incompressible fluid [2] it is known that the momentum thickness $\delta^{**}(x)$ as well as the displacement thickness $\delta^*(x)$ are determined with the following expressions in the form of integrals:

$$\delta^*(x)=\int_0^\infty(1-\frac{u}{u_e})\,\mathrm{d}y,\ \ \delta^{**}(x)=\int_0^\infty\frac{u}{u_e}(1-\frac{u}{u_e})\mathrm{d}y . \tag{6}$$

These thicknesses refer to the incompressible fluid flow, for which the density $\rho=\mathrm{const}$.

Note that the integral in the expression for I_1, which regards the compressible fluid, i.e. the dissociated gas, is similar in the form to the integral (6) which defines the momentum thickness of an incompressible fluid. Since the density of compressible fluid is the function of two variables, i.e. $\rho=\rho(x,y)$; the density ρ can be eliminated from sub-integral function of the integral I_1 with a suitable transformation of variables, so that two integrals can be made formally identical. That is why it is necessary to transform the physical coordinate y into a new transversal variable for all values of the longitudinal variable x in the boundary layer. If we compare I_1 and δ^{**} we can conclude that the necessary transformation of the variables must satisfy the relation

$$\rho(x,y)\,\mathrm{d}y=\rho_0\,\mathrm{d}z . \tag{7}$$

In the relation (7), z denotes the new transversal variable, while $\rho_0=\mathrm{const.}$ denotes an arbitrary known value of the compressible fluid density. Since $\rho_0=\mathrm{const.}$, it follows from (7) that the newly introduced transversal variable z must be a function of both x and y, i.e. $z=z(x,y)$. The solution of the differential equation (7), i.e. transformation of the variables in the form of this relation should ensure that in any cross-section of the boundary layer, i.e. for all the values of the longitudinal variable x: $\mathrm{d}z/\mathrm{d}y=\rho/\rho_0$, or to be more precise

$$\frac{\partial z}{\partial y}=\frac{\rho}{\rho_0} . \tag{8}$$

Since limit values in the expressions for the integrals I_1 and δ^{**} are the same, it is necessary that the physical transversal variable y and the newly introduced transversal variable $z(x,y)$ should have the same values at the edges of the boundary layer.

With these necessary conditions, I_1 can be brought to this form

$$I_1 = \frac{\mathrm{d}}{\mathrm{d}x}\left[u_e^2 \int\limits_0^\infty \frac{u}{u_e}(1-\frac{u}{u_e})\rho_0 \mathrm{d}z\right] =$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}\left[\rho_0 u_e^2 \int\limits_0^\infty \frac{u}{u_e}(1-\frac{u}{u_e})\,\mathrm{d}z\right] = \frac{\mathrm{d}}{\mathrm{d}x}(\rho_0 u_e^2 \Delta^{**})$$

where Δ^{**} stands for the integral

$$\Delta^{**} = \int\limits_0^\infty \frac{u}{u_e}(1-\frac{u}{u_e})\,\mathrm{d}z = \Delta^{**}(x). \tag{9}$$

It is clear that the expressions for the value Δ^{**} and for the momentum thickness δ^{**} are of the same form. That is why, by analogy with δ^{**}, the value Δ^{**} is, in the literature [6], called the conditional displacement thickness.

According to the given relations and conditions of "norm setting", the term I_3 of the equation (4) can be transformed as

$$I_3 = \frac{\mathrm{d}u_e}{\mathrm{d}x}\int\limits_0^\infty (\rho u - \rho_e u_e)\,\mathrm{d}y = \text{-}u_e \frac{\mathrm{d}u_e}{\mathrm{d}x}\int\limits_0^\infty \rho(\frac{\rho_e}{\rho} - \frac{u}{u_e})\,\mathrm{d}y =$$

$$= \text{-}u_e \frac{\mathrm{d}u_e}{\mathrm{d}x}\int\limits_0^\infty (\frac{\rho_e}{\rho} - \frac{u}{u_e})\rho_0 \mathrm{d}z = \text{-}\rho_0 u_e \Delta^* \frac{\mathrm{d}u_e}{\mathrm{d}x} ,$$

where

$$\Delta^* = \int\limits_0^\infty (\frac{\rho_e}{\rho} - \frac{u}{u_e})\,\mathrm{d}z = \Delta^*(x) \tag{10}$$

Because of the formal similarity of the expression (10) for the value Δ^* with the expression (6), which defines the displacement thickness of the incompressible fluid δ^*, this value is, in the literature, called the conditional displacement thickness.

Based on the analysis undertaken here, and from the condition of the formal identity of the considered integrals, it follows, according to (8), that the transformation of the transversal variable in the form of the expression

$$\boxed{z(x,y) = \int\limits_0^y \frac{\rho}{\rho_0}\mathrm{d}y} \tag{11}$$

satisfies all in advance given conditions of the norm-setting.

By means of the newly introduced transversal variable (11), Eq. (4) of compressible fluid transforms into

$$\frac{\mathrm{d}}{\mathrm{d}x}(u_e^2 \Delta^{**}) + u_e \Delta^* \frac{\mathrm{d}u_e}{\mathrm{d}x} = \frac{\tau_w}{\rho_0} + \frac{\rho_w}{\rho_0} v_w u_e$$

and after differentiation we come to the nondimensional equation which is

$$\frac{\mathrm{d}\Delta^{**}}{\mathrm{d}x} + \frac{\mathrm{d}u_e}{\mathrm{d}x}\frac{\Delta^{**}}{u_e}(2+H) = \frac{\tau_w}{\rho_0 u_e^2} + \frac{\rho_w}{\rho_0}\frac{v_w}{u_e}, \tag{12}$$

where

$$H = \frac{\Delta^*}{\Delta^{**}} .$$

This way the obtained momentum equation of the dissociated gas (12) has almost identical form with the momentum equation of the incompressible fluid for the case of the flow along the porous wall of the body within the fluid [2, 9]. Besides, for the case $\rho = \rho_0 = \rho_w = \rho_e = \mathrm{const.}$, the newly introduced transversal variable comes down to $z(x,y) = y$, the conditional thicknesses of the boundary layer reduce to $\Delta^{**} = \delta^{**}$, $\Delta^* = \delta^*$, while the momentum equation (12) transforms to the momentum equation of the incompressible fluid for the corresponding flow problem.

With the newly introduced transversal variable $z(x,y)$, the shear stress on the wall of the body within the fluid can be written in the form of the expression

$$\tau_w = \frac{\rho_w \mu_w}{\rho_0}\frac{u_e}{\Delta^{**}}\zeta , \tag{13}$$

where the nondimensional friction function ζ is defined as

$$\zeta = \left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})}\right]_{z=0} = \zeta(x) . \tag{14}$$

Substituting the expression (13) into the momentum equation (12) we obtain Eq.

$$\frac{\mathrm{d}\Delta^{**}}{\mathrm{d}x} + \frac{\mathrm{d}u_e}{\mathrm{d}x}\frac{\Delta^{**}}{u_e}(2+H) = \frac{\rho_w \mu_w}{\rho_0^2}\frac{\zeta}{u_e \Delta^{**}} + \frac{\rho_w}{\rho_0}\frac{v_w}{u_e} .$$

After some transformations the previous equation can be brought to the equation of the form

$$\frac{\mathrm{d}Z^{**}}{\mathrm{d}x} = \frac{\overline{F}_{dp}}{u_e} , \tag{15}$$

and as such it is usually stated in the literature for different cases of the fluid flow. In Eq. (15) the characteristic function $\overline{F}_{dp}$ of the dissociated gas boundary layer is determined with the expression

$$\overline{F}_{dp} = 2\left[\frac{\rho_w \mu_w}{\rho_0 \mu_0}\zeta - (2+H)\,\overline{f}\right] + 2\frac{\rho_w v_w \Delta^{**}}{\rho_0 \nu_0} , \tag{16}$$

$$(\ \nu_0 = \mu_0 / \rho_0\ )$$

where the function Z^{**} and the so-called parameter of the form $\overline{f}$ are determined as:

$$Z^{**} = \frac{\Delta^{**2}}{\nu_0} = Z^{**}(x) \ , \quad \frac{\mathrm{d}u_e}{\mathrm{d}x}Z^{**} = \frac{\mathrm{d}u_e}{\mathrm{d}x}\frac{\Delta^{**2}}{\nu_0} = \overline{f}(x) .$$

The corresponding momentum equation of incompressible fluid [2] is of the same form as Eq. (15). However, the characteristic function of the boundary layer F_p is, in that case, determined with the expression

$$F_p = 2\left[\zeta_t - (2+H_t) f_t\right] - 2\lambda \ , \qquad (17)$$

where for incompressible fluid, we use the subscript t

$$\zeta_t = \left[\frac{\partial(u/u_e)}{\partial(y/\delta^{**})}\right]_{y=0} \ , \quad f_t = \frac{\mathrm{d}u_e}{\mathrm{d}x}\frac{\delta^{**2}}{\nu} \ ,$$

$$H_t = \frac{\delta^*}{\delta^{**}} \ , \ Z_t = \frac{\delta^{**2}}{\nu} \ , \ \lambda = -\frac{v_w \delta^{**}}{\nu} \ .$$

In the given expressions λ is the porosity parameter. For the case of a non-porous wall of the body within the fluid, the porosity parameter equals zero, because $v_w(x) = 0$.

Because of the methodology of the formal identity of the corresponding values with compressible and incompressible fluid, a request of formal identity of the characteristic functions $\bar{F}_{dp}$ and F_p can be made. The "coefficient" next to the nondimensional function ζ in the relation (16) indicates that a new longitudinal variable of the boundary layer should be introduced instead of the physical coordinate x. The new longitudinal variable is, for now, defined with the general relation

$$s = s(x).$$

By means of the newly introduced longitudinal variable ($u_e(x) \to u_e(s)$, $Z^{**}(x) \to Z^{**}(s), ...$) and the transformation of the differentiation, the parameter of the form $\bar{f}$ comes down to

$$\bar{f}(x) = \frac{\mathrm{d}u_e}{\mathrm{d}x}\frac{\Delta^{**2}(x)}{\nu_0} = \frac{\mathrm{d}u_e}{\mathrm{d}s}\frac{\Delta^{**2}}{\nu_0}\frac{ds}{dx} = f\frac{\mathrm{d}s}{\mathrm{d}x}$$

where now we have

$$f = \frac{\mathrm{d}u_e}{\mathrm{d}s}\frac{\Delta^{**2}}{\nu_0} = \frac{u_e' \Delta^{**2}}{\nu_0} = f(s) \ . \qquad (18)$$

The momentum equation (15) is, in this case, transformed into the equation

$$\frac{\mathrm{d}Z^{**}}{\mathrm{d}s} = \frac{F_{dp}}{u_e} \ , \qquad (19)$$

in which the characteristic function F_{dp} is determined with the expression

$$F_{dp} = \frac{1}{\mathrm{d}s/\mathrm{d}x}\bar{F}_{dp} =$$

$$= 2\left[\zeta\frac{\rho_w\mu_w}{\rho_0\mu_0}\frac{1}{\mathrm{d}s/\mathrm{d}x} - (2+H)f\right] + 2\frac{\rho_w v_w \Delta^{**}}{\rho_0 \nu_0}\frac{1}{\mathrm{d}s/\mathrm{d}x}$$

Having this form of the expression F_{dp}, it is obvious that with the relation

$$\boxed{s(x) = \frac{1}{\rho_0\mu_0}\int_0^x \rho_w\mu_w \mathrm{d}x} \ , \qquad (20)$$

$$\left(\frac{\mathrm{d}s}{\mathrm{d}x} = \frac{\rho_w}{\rho_0}\frac{\mu_w}{\mu_0}\right) ,$$

the function F_{dp} becomes

$$F_{dp} = 2[\zeta - (2+H)f] + 2\frac{v_w\Delta^{**}}{\nu_0}\frac{\mu_0}{\mu_w}$$

In the expression (20) for the newly introduced longitudinal variable $s(x)$, the values ρ_w and μ_w stand for the known distributions of the density and dynamic viscosity at the wall of the body within the fluid.

If, with the compressible fluid flow, we introduce the porosity parameter as

$$\Lambda(s) = -\frac{v_w\Delta^{**}}{\nu_0}\frac{\mu_0}{\mu_w} \ , \qquad (21)$$

then for the characteristic function F_{dp} of the boundary layer of the dissociated gas we get the expression which takes its final form as

$$F_{dp} = 2[\zeta - (2+H)f] - 2\Lambda \ . \qquad (22)$$

The obtained expression (22) for the characteristic function F_{dp} of the dissociated gas boundary layer is, with the considered conditions, of the same form as the corresponding expression (17) for the characteristic function F_p of incompressible fluid. For the case of the boundary layer flow along the non-porous wall ($v_w = 0$) the expressions for the both characteristic functions F_{dp} and F_p formally reduce to the familiar expression [2] characteristic for the incompressible fluid flow

$$F = 2[\ \zeta - (2+H)f] \ .$$

3. CONCLUDING DISCUSSION

Therefore, by a detailed analysis undertaken here, we present a natural procedure to obtain the form of the transformation of the transversal and longitudinal variables of the boundary layer. At the same time, the answer to the question concerned with the purpose of the newly introduced variables $s(x)$ and $z(x, y)$ was given. By introducing these variables, we achieve that the boundary layer expressions of the compressible fluid (dissociated gas) for the conditional thicknesses, for the momentum equation and for the characteristic function F_{dp} have *the same form* as the corresponding expressions for the case of the incompressible flow. In addition, for the conditions of isothermal liquid flow across the porous wall ($\rho = \text{const.}$, $\mu = \text{const.}$), these values of the dissociated gas come down to the corresponding values of incompressible fluid. In this case the porosity parameter of the dissociated gas $\Lambda(s)$ transforms to the porosity parameter of incompressible fluid $\lambda(x)$. Consequently the corresponding sets of parameters, upon which the General Similarity Method is based [5], are determined with the formally identical formulas [2]. This way the application and control of obtaining so-called generalized equations with the General Similarity Method is much easier for different cases of the compressible fluid flow in the boundary layer.

Moreover, the transformations of the variables (11) and (20), i.e. transformations (1), were known in the literature as Dorodnicin's transformations modified by Lees L. They were particularly used for investigation of the dissociated and ionized gas flow in the boundary layer [6, 7]. Since this paper shows a logical procedure to obtain them and pinpoints the benefit of their usage, it is *important*, first of all, *from a methodological point of view*.

At the end of this analysis, it should be once more stated that the introduction of the new variables $s(x)$ and $z(x, y)$ in the form of the transformations (20) and (11) was determined based on the condition of the formal identity of the momentum equations and the characteristic functions with compressible and incompressible fluid.

Thus defined transformations of the variables should be applied to the dynamic equation of the system (2). Here, as with other fluid flow problems, the stream function $\psi(s, z)$ is introduced. This function is introduced in accordance with

$$u = \frac{\partial \psi}{\partial z} \ , \ \ \tilde{v} = \frac{\rho_0}{\rho_w} \frac{\mu_0}{\mu_w} \left(u \frac{\partial z}{\partial x} + v \frac{\rho}{\rho_0} \right) = -\frac{\partial \psi}{\partial s} \ , \qquad (23)$$

which is based on the momentum equation of the considered problem of the compressible fluid flow.

After the transformation of the variables and substituting (23) the starting dynamic equation of the system (2) becomes

$$\begin{gathered} \frac{\partial \psi}{\partial z} \frac{\partial^2 \psi}{\partial s \partial z} - \frac{\partial \psi}{\partial s} \frac{\partial^2 \psi}{\partial z^2} = \frac{\rho_e}{\rho} u_e \frac{\mathrm{d} u_e}{\mathrm{d} s} + \nu_0 \frac{\partial}{\partial z} \left(Q \frac{\partial^2 \psi}{\partial z^2} \right) , \\ z = 0 : \quad \frac{\partial \psi}{\partial z} = 0 \ , \quad \frac{\partial \psi}{\partial s} = -\frac{\mu_0}{\mu_w} v_w \ , \qquad (24) \\ z \to \infty : \quad \frac{\partial \psi}{\partial z} \to u_e(s) . \end{gathered}$$

The nondimensional function Q in the obtained equation is determined with the expression

$$Q = \frac{\rho}{\rho_w} \frac{\mu}{\mu_w}$$

Obviously, $Q = 1$ for $z = 0$, and

$$Q = \frac{\rho_e}{\rho_w} \frac{\mu_e}{\mu_w}$$

for $z \to \infty$. If the obtained Eq. (24) is compared to the corresponding known equation for the case of the incompressible fluid flow [2, 9] expressed by means of the stream function $\psi(x, y)$,

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = u_e \frac{\mathrm{d} u_e}{\mathrm{d} x} + \nu \frac{\partial^3 \psi}{\partial y^3} \ , \qquad (25)$$

it can be seen that these two equations are almost of the same form. To be more precise, the left-hand sides of Eqs. (24) and (25) have the same form. The right-hand sides do not. However, in the case of the isothermal flow of incompressible fluid ($\rho = \text{const.}$, $\mu = \text{const.}$), the function Q becomes equal to 1, while Eq. (24) basically transforms to the corresponding Eq. (25) of the incompressible fluid. Under these conditions these two equations are identical.

Therefore, by application of the new variables $s(x)$ and $z(x, y)$ in the form of the transformations (1), the dynamic boundary layer equation of compressible fluid is brought to almost (i.e. partially) identical form as the corresponding boundary layer equation of incompressible fluid.

Note that, under the same conditions [2], by means of Stewartson's variables, the dynamic equation of the system (2) can be brought to the completely same form as the corresponding equation of incompressible fluid.

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О ТРАНСФОРМАЦИЈАМА ПРОМЕНЉИВИХ КОД РЕШАВАЊА ЈЕДНАЧИНА ГРАНИЧНОГ СЛОЈА СТИШЉИВОГ ФЛУИДА

Бранко Р. Обровић, Слободан Р. Савић

Овај рад се односи на струјање стишљивог флуида, односно дисоцираног гаса у граничном слоју. У раду су на један логични начин дефинисане трансформације подужне и попречне променљиве граничног слоја, које су већ од раније познате у литератури. Поменуте трансформације се користе као претходне код примене Методе уопштене сличности за решавање једначина граничног слоја стишљивог флуида.

У раду је показано да се ове трансформације могу дефинисати из услова истоветности облика одговарајућих величина и једначина граничног слоја стишљивог и нестишљивог флуида.